WI3417TU
Kansmodellen voor Finance
Assignment 9

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Chapter 4, Exercise 5

All stopping times in $\mathcal{S}_0$:

1. $\tau(HH) = \tau(HT) = \tau(TH) = \tau(TT) = 0$
2. $\tau(HH) = \tau(HT) = 1, \tau(TH) = \tau(TT) = 1$
3. $\tau(HH) = \tau(HT) = 2, \tau(TH) = \tau(TT) = 1$
4. $\tau(HH) = \tau(HT) = 1, \tau(TH) = \tau(TT) = 2$
5. $\tau(HH) = \tau(HT) = 2, \tau(TH) = \tau(TT) = 2$
6. $\tau(HH) = \tau(HT) = \infty, \tau(TH) = \tau(TT) = 1$
7. $\tau(HH) = \tau(HT) = \infty, \tau(TH) = \tau(TT) = 2$
8. $\tau(HH) = \tau(HT) = 1, \tau(TH) = \tau(TT) = \infty$
9. $\tau(HH) = \tau(HT) = 2, \tau(TH) = \tau(TT) = \infty$
10. $\tau(HH) = \tau(HT) = \infty, \tau(TH) = \tau(TT) = \infty$
11. $\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = \tau(TT) = 1$
12. $\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = \tau(TT) = 2$
13. $\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = \tau(TT) = \infty$
14. $\tau(HH) = 2, \tau(HT) = \infty, \tau(TH) = \tau(TT) = 1$
15. $\tau(HH) = 2, \tau(HT) = \infty, \tau(TH) = \tau(TT) = 2$
16. $\tau(HH) = 2, \tau(HT) = \infty, \tau(TH) = \tau(TT) = \infty$
17. $\tau(HH) = \tau(HT) = 1, \tau(TH) = \infty, \tau(TT) = 2$
18. $\tau(HH) = \tau(HT) = 2, \tau(TH) = \infty, \tau(TT) = 2$
19. $\tau(HH) = \tau(HT) = \infty, \tau(TH) = \infty, \tau(TT) = 2$

20. $\tau(HH) = \tau(HT) = 1, \tau(TH) = 2, \tau(TT) = \infty$

21. $\tau(HH) = \tau(HT) = 2, \tau(TH) = 2, \tau(TT) = \infty$

22. $\tau(HH) = \tau(HT) = \infty, \tau(TH) = 2, \tau(TT) = \infty$

23. $\tau(HH) = 2, \tau(HT) = \infty, \tau(TH) = 2, \tau(TT) = \infty$

24. $\tau(HH) = 2, \tau(HT) = \infty, \tau(TH) = \infty, \tau(TT) = 2$

25. $\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = \infty, \tau(TT) = 2$

26. $\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = 2, \tau(TT) = \infty$

Stopping times that do not exercise out-of-the-money, and their accompanying values:

$\tau(HH) = \tau(HT) = \tau(TH) = \tau(TT) = 0$	1
$\tau(HH) = \tau(HT) = \infty, \tau(TH) = \tau(TT) = 1$	1.2
$\tau(HH) = \tau(HT) = \infty, \tau(TH) = \tau(TT) = 2$	0.8
$\tau(HH) = \tau(HT) = \infty, \tau(TH) = \tau(TT) = \infty$	0.0
$\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = \tau(TT) = 1$	1.36
$\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = \tau(TT) = 2$	0.96
$\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = \tau(TT) = \infty$	0.16
$\tau(HH) = \tau(HT) = \infty, \tau(TH) = \infty, \tau(TT) = 2$	0.64
$\tau(HH) = \tau(HT) = \infty, \tau(TH) = 2, \tau(TT) = \infty$	0.16
$\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = \infty, \tau(TT) = 2$	0.8
$\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = 2, \tau(TT) = \infty$	0.32

The stopping time at (4.4.6) indeed gives the highest value, 1.36, here.

Chapter 4, Exercise 6

i

For all $\tau \in \mathcal{S}_0, \tau \leq N$:

$$\tilde{\mathbb{E}}\left[\frac{K - S_\tau}{(1+r)^\tau}\right] = \frac{K}{(1+r)^\tau} - \tilde{\mathbb{E}}\left[\frac{S_\tau}{(1+r)^\tau}\right]$$

Because the discounted stock price is a martingale, for any n:

$$S_0 = \tilde{\mathbb{E}}\left[\frac{S_n}{(1+r)^n}\right]$$

Therefore:

$$\tilde{\mathbb{E}}\left[\frac{K - S_\tau}{(1+r)^\tau}\right] = \frac{K}{(1+r)^\tau} - S_0$$

This is clearly maximal when $(1+r)^\tau$ is minimal. Since r is positive, that is minimal when $\tau = 0$. The optimal exercise policyt is thus to sell the stock at time zero, at which point it takes the value $K - S_0$.

ii

If the put was at any point more valuable, we could sell an american put, then buy the european call and the security from (i) and have money left.

If the put is ever exercised, immediately exercise the security from (i) to make back the money spent, then let the european call expire without exercise.

If the put is not exercised, at time N exercise both the security from (i) and the european call. This gives a net change of 0.

This is arbitrage, because we are making more money from the initial sale, but never have to spend any extra. The american put, therefore, cannot be worth more at any time.

Because this is true at any time, including time 0, we could also say that:

$$V_0^{AP} \leq V_0^{(i)} + V_0^{EC}$$
$$V_0^{AP} \leq K - S_0 + V_0^{EC}$$

iii

An american put that is exercised only at time N or never gives exactly the same payoff as a european put, so an american put cannot be worth less than a european put, because the optimal exercise policy gives at least as great an expected payoff as the european put's "exercise policy".

So:

$$V_0^{EP} \leq V_0^{AP}$$

Put-call parity states that:

$$V_0^{EC} = V_0^{EP} + S_0 - \frac{K}{(1+r)^N}$$

And thus:

$$\frac{K}{(1+r)^N} - S_0 + V_0^{EC} \leq V_0^{AP}$$