

WI3417TU
Kansmodellen voor Finance
Assignment 6

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Chapter 3, Exercise 1

(iii')

$$\begin{aligned}\mathbb{E}Y &= \sum_{w \in \Omega} Y(w)\mathbb{P}(w) \\ &= \sum_{w \in \Omega} Y(w) \frac{\mathbb{P}(w)}{\tilde{\mathbb{P}}(w)} \tilde{\mathbb{P}}(w) \\ &= \sum_{w \in \Omega} Y(w) \frac{1}{Z(w)} \tilde{\mathbb{P}}(w) \\ &= \tilde{\mathbb{E}} \left[Y \frac{1}{Z} \right]\end{aligned}$$

Chapter 3, Exercise 4

(i)

$$\begin{aligned}\zeta_3(HHH) &= \frac{27}{64} * \frac{1}{1.25^3} = 0.216 \\ \zeta_3(HHT) &= \frac{27}{32} * \frac{1}{1.25^3} = 0.432 \\ \zeta_3(HTT) &= \frac{27}{16} * \frac{1}{1.25^3} = 0.864 \\ \zeta_3(TTT) &= \frac{27}{8} * \frac{1}{1.25^3} = 1.728\end{aligned}$$

(ii)

$$\begin{aligned}V_0 &= \sum_{w \in \Omega} V_N(w) \zeta(w) \mathbb{P}(w) \\ &= \mathbb{P}(HHH) * V_N(HHH) * \zeta(HHH) + \dots \\ &= 1.216\end{aligned}$$

(iii)

$$\begin{aligned}\zeta_2(HH) &= \frac{9}{8} * \frac{1}{1.25^2} \\ &= 0.72\end{aligned}$$

(iv)

$$\begin{aligned}V_2(HH) &= \frac{1}{0.72} \mathbb{E}_2[\zeta_3 V_3](HH) \\ &= \frac{1}{0.72} \left(\frac{2}{3} * 0.432 * 2 + \frac{1}{3} * 0.864 * 0.5 \right) \\ &= 1\end{aligned}$$

$$\begin{aligned}V_2(TH) &= \frac{1}{0.72} \mathbb{E}_2[\zeta_3 V_3](TH) \\ &= \frac{1}{0.72} \left(\frac{2}{3} * 0.432 * 0.5 + \frac{1}{3} * 0.864 * 0 \right) \\ &= 0.2\end{aligned}$$

Additional Exercise

(i)

$$\begin{aligned}u_0 &= 1.1 \\d_0 &= 0.7 \\ \tilde{p}_0 &= 0.75 \\ \tilde{q}_0 &= 0.25\end{aligned}$$

$$\begin{aligned}u_1(H) &= \frac{13}{11} \\d_1(H) &= \frac{9}{11} \\ \tilde{p}_1(H) &= 0.5 \\ \tilde{q}_1(H) &= 0.5\end{aligned}$$

$$\begin{aligned}u_1(T) &= \frac{8}{7} \\d_1(T) &= \frac{6}{7} \\ \tilde{p}_1(T) &= 0.5 \\ \tilde{q}_1(T) &= 0.5\end{aligned}$$

(ii)

$$V_1(H) = 2 + 0.5 * \frac{7}{3} + 0.5 * 1 = \frac{11}{3}$$

$$V_1(T) = 0.5 * 0 + 0.5 * 0 = 0$$

$$V_0 = 0.75 * V_1(H) + 0.25 * V_1(T) = 2.75$$

(iii)

$$\Delta_1(H) = \frac{V_2(HH) - V_2(HT)}{S_2(HH) - S_2(HT)} = \frac{\frac{4}{3}}{4} = \frac{1}{3}$$

$$\Delta_1(T) = \frac{V_2(TH) - V_2(TT)}{S_2(TH) - S_2(TT)} = \frac{0}{2} = 0$$

$$\Delta_0 = \frac{V_1(H) - V_1(T)}{S_1(H) - S_1(T)} = \frac{\frac{11}{3}}{4} = \frac{11}{12}$$