WI3411TU
Monte Carlo Methods
Huiswerk 5

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P21.1

```
% Average price asian call
% Uses Monte Carlo with antithetic variates

randn('state',1337)

%%%%%%%%% Problem and method parameters %%%%%%%%%%
S = 5; E = 6; sigma = 0.3; r = 0.05; T = 1; B = 9;
Dt = 1e-3; N = T/Dt; M = 1e4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

V = zeros(M,1);
Vanti = zeros(M,1);
for i = 1:M
    samples = randn(N,1);

    % standard Monte Carlo
    Svals = S*cumprod(exp((r-0.5*sigma^2)*Dt+sigma*sqrt(Dt)*samples));
    V(i) = exp(-r*T)*max(mean(Svals)-E,0);

    % antithetic path
    Svals2 = S*cumprod(exp((r-0.5*sigma^2)*Dt-sigma*sqrt(Dt)*samples));
    V2 = exp(-r*T)*max(mean(Svals2)-E,0);
    Vanti(i) = 0.5*(V(i) + V2);

end
aM = mean(V)
bM = std(V)
se = bM/sqrt(M)
conf = [aM - 1.96*bM/sqrt(M), aM + 1.96*bM/sqrt(M)]
vM = var(V)

aManti = mean(Vanti)
bManti = std(Vanti)
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seAnti = bManti/sqrt(M)
confanti = [aManti - 1.96*bManti/sqrt(M), aManti + 1.96*bManti/sqrt(M)]
vManti = var(Vanti)

ratio = vM/vManti
```

De schatting met gewone monte carlo is $0.0981 \pm 0.0032(s.e.)$. De schatting van het antithetic model is $0.0944 \pm 0.0021(s.e.)$.

De verkregen ratio tussen de varianties van de datasets is 2.3156. De trekkingen kosten dubbel zo veel werk, dus de totale verbetering in efficientie is $\frac{2.2847}{2} = 1.1578$.

De schatting op Dt=0.001 is aManti = 0.0944. Op Dt=0.1 is aManti = 0.1164. De bias zal dus positief zijn en rond de 0.022 liggen.

P22.1

```
%CH22       Program for Chapter 22
%
% Monte Carlo on an arithmetic average price Asian option
% using a geometric average price Asian as control variate

randn('state',1337)

%%%%%% Problem and method parameters %%%%%%%%
S = 4; E = 4; sigma = 0.25; r = 0.03; T = 1;
Dt = 1e-2; N = T/Dt; M = 1e4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%% Geom Asian exact mean  %%%%%%%%%%%
sigsqT= sigma^2*T*(N+1)*(2*N+1)/(6*N*N);
muT = 0.5*sigsqT + (r - 0.5*sigma^2)*T*(N+1)/(2*N);

d1 = (log(S/E) + (muT + 0.5*sigsqT))/(sqrt(sigsqT));
d2 = d1 - sqrt(sigsqT);

N1 = 0.5*(1+erf(d1/sqrt(2)));
N2 = 0.5*(1+erf(d2/sqrt(2)));

geo =  exp(-r*T)*( S*exp(muT)*N1 - E*N2 );
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Spath = S*cumprod(exp((r-0.5*sigma^2)*Dt+sigma*sqrt(Dt)*randn(M,N)),2);

% Standard Monte Carlo
arithave = mean(Spath,2);
Parith = exp(-r*T)*max(arithave-E,0);  % payoffs
Pmean = mean(Parith)
Pstd = std(Parith)
Pse = Pstd/sqrt(M)
Pvar = var(Parith)
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confmc = [Pmean-1.96*Pstd/sqrt(M), Pmean+1.96*Pstd/sqrt(M)]

% Control Variate
geoave = exp((1/N)*sum(log(Spath),2));
Pgeo = exp(-r*T)*max(geoave-E,0);      % geo payoffs

%Calculate theta
covar = cov(Parith, Pgeo);
theta = covar(1, 2) / var(Pgeo)

Z = Parith + (geo - Pgeo) * theta;     % control variate version
Zmean = mean(Z)
Zstd = std(Z)
Zse = Zstd/sqrt(M)
Zvar = var(Z)
confcv = [Zmean-1.96*Zstd/sqrt(M), Zmean+1.96*Zstd/sqrt(M)]

ratio=Pvar/Zvar
```

De gebruikte theta is 1.0427.

De schatting zonder control variate is $0.2546 \pm 0.0039(s.e.)$.

De schatting met control variate is $0.25876 \pm 0.00013(s.e.)$.

De ratio tussen de varianties is 915.4. Als we er van uit gaan dat de control variate methode op zijn meest 2x zo veer werk kost, dan is deze dus op zijn minst 457.7 keer zo snel.