WI3411TU
Monte Carlo Methods
Huiswerk 2

Lucas de Vries 1522442

29 november 2011

MIPS 6.8

We need to find $F^{inv}(u)$ by solving for x in $F(x) = u$.

$$\begin{aligned} 1 - x^{-3} &= u \\ x^{-3} &= 1 - u \\ x &= \frac{1}{\sqrt[3]{1-u}} \end{aligned}$$

So we can construct X from a U(0, 1) through $X = \frac{1}{\sqrt[3]{1-U}}$.

Higham 4.3

The distribution function evaluates to the current quantile at a specific value, so the inverse of the distribution function maps quantiles to values.

That is:

$$\begin{aligned} N(x) &= p \\ \frac{1 + erf\left(x/\sqrt{2}\right)}{2} &= p \\ erf\left(x/\sqrt{2}\right) &= 2p - 1 \\ x/\sqrt{2} &= \text{inverf}(2p-1) \\ x &= \sqrt{2}\text{inverf}(2p-1) \end{aligned}$$

We know about the distribution function F for an N(3, 5) distribution that:

$$F(x) = \sqrt{5}N(x) + 3$$

So that:

$$\begin{aligned}
\sqrt{5}N(x) + 3 &= u \\
N(x) &= \frac{u-3}{\sqrt{5}} \\
x &= N^{inv}\left(\frac{u-3}{\sqrt{5}}\right) \\
F^{inv}(u) &= \sqrt{2}\text{inverf}\left(2\frac{u-3}{\sqrt{5}} - 1\right)
\end{aligned}$$

Filling in the results from our U(0, 1) sample into $F^{inv}(U)$ simulates an N(3, 5) variable.

Higham P4.1

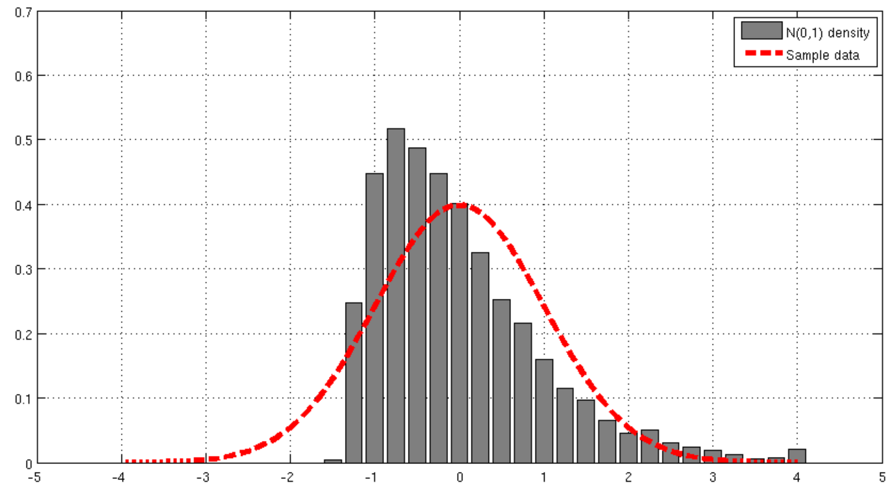
Program:

```
%CH04     Program for Chapter 4
%
% Histogram illustration of Central Limit Theorem

clf
colormap([0.5 0.5 0.5])
rand('state',100)

n = 2;
M = 1e+4;

mu = 1;
sigma = 1;

S = zeros(M,1);
for k = 1:M
     S(k) = (sum(-log((rand(n,1)))) - n*mu)/(sigma*sqrt(n));
end

%%%%%%%%%%%%%%%% Histogram %%%%%%%%%%%%%%%%%%
dx = 0.25;
centers = [-4:dx:4];
N = hist(S,centers);
bar(centers,N/(M*dx))
```

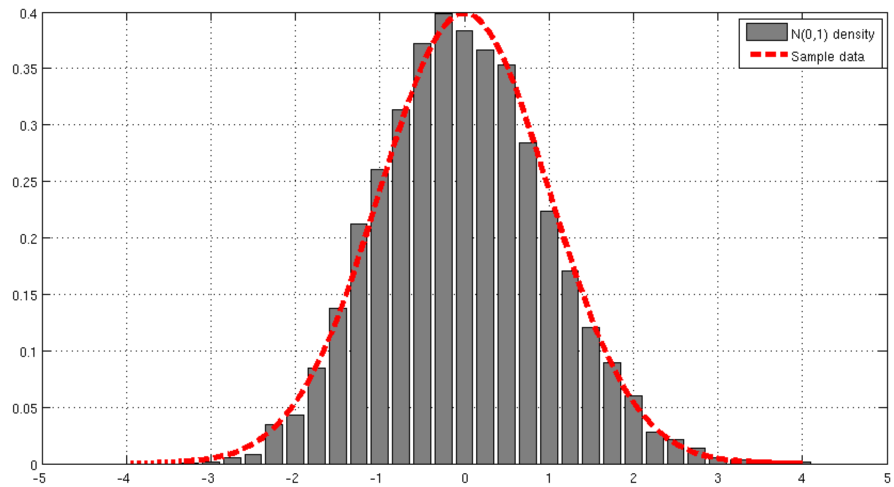
```
hold on
x = linspace(-4,4,100);
y = exp(-0.5*x.^2)/sqrt(2*pi);
plot(x,y,'r--','Linewidth',4)
legend('N(0,1) density','Sample data')
grid on
```

Graphs:

$n = 2$:



$n = 100$:



$n = 1000$: