WI3417TU
Kansmodellen voor Finance
Assignment 10

Lucas de Vries 1522442

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Chapter 5, Exercise 1

(i)

Because $\tau_2 - \tau_1$ has the same distribution as τ_1:

$$\begin{aligned}
(\mathbb{E}\alpha^{\tau_1})^2 &= \mathbb{E}\alpha^{\tau_1}\mathbb{E}\alpha^{\tau_2-\tau_1} \\
&= \mathbb{E}\alpha^{\tau_1}\alpha^{\tau_2-\tau_1} && \text{(Independence)} \\
&= \mathbb{E}\alpha^{\tau_2}
\end{aligned}$$

(ii)

Assume that the following equation holds for some n:

$$(\mathbb{E}\alpha^{\tau_1})^n = \mathbb{E}\alpha^{\tau_n}$$

Then it must also hold for n+1, following:

$$\begin{aligned}
(\mathbb{E}\alpha^{\tau_1})^{n+1} &= \mathbb{E}\alpha^{\tau_{n+1}-\tau_n}(\mathbb{E}\alpha^{\tau_1})^n && (\tau_{n+1} - \tau_n \text{ has the same distribution as } \tau_1) \\
&= \mathbb{E}\alpha^{\tau_{n+1}-\tau_n}\mathbb{E}\alpha^{\tau_n} && \text{(Assumption)} \\
&= \mathbb{E}\alpha^{\tau_{n+1}-\tau_n}\alpha^{\tau_n} && \text{(Independence)} \\
&= \mathbb{E}\alpha^{\tau_{n+1}}
\end{aligned}$$

Because we know from (i) that it holds for $n = 2$ (and $n = 1$ is a trivial case), we have proven by induction that (5.7.1) holds for all positive integers n.

(iii)

The 'facts' presented in the exercise still hold for asymmetric random walks: $\tau_2 - \tau_1$ is still independent of τ_1, because they depend on completely separate sequences of coin tosses. Furthermore, $\tau_2 - \tau_1$ is still identically distributed to τ_1: both represent the amount of steps needed to get from a starting level to 1 higher than that starting level. The exact starting level of a random walk does not change the expected time required to reach an offset of the starting level.

Because the same assumptions still hold, the same conclusions still hold as well, so (5.7.1) holds for assymetric random walks too.

Intuitively, this makes sense: the chance to independently go up by one twice is equal to the chance to go up by one once multiplied by itself. The exact probabilities involved do not matter for applying this multiplication.

Chapter 5, Exercise 4

(i)

Because the random walk can reach level 2 only on an even-numbered step, we can rewrite 5.2.18 as:

$$\begin{aligned}\mathbb{E}\alpha^{\tau_2} &= \sum_{k=1}^{\infty} \alpha^{2k}\mathbb{P}\{\tau_2 = 2k\} \\ &= \left(\frac{1-\sqrt{1-\alpha^2}}{\alpha}\right)^2\end{aligned}$$

So from the exercise:

$$\sum_{k=1}^{\infty} \alpha^{2k}\mathbb{P}\{\tau_2 = 2k\} = \sum_{k=1}^{\infty} \left(\frac{\alpha}{2}\right)^{2k} \frac{(2k)!}{(k+1)!k!}$$

Equating termwise:

$$\begin{aligned}\alpha^{2k}\mathbb{P}\{\tau_2 = 2k\} &= \left(\frac{\alpha}{2}\right)^{2k} \frac{(2k)!}{(k+1)!k!} \\ \mathbb{P}\{\tau_2 = 2k\} &= \left(\frac{1}{2}\right)^{2k} \frac{(2k)!}{(k+1)!k!}\end{aligned}$$